Leggett modes and the Anderson–Higgs mechanism in superconductors without inversion symmetry

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We develop a microscopic and gauge–invariant theory for collective modes resulting from the phase of the superconducting order parameter in non–centrosymmetric superconductors. Considering various crystal symmetries we derive the corresponding gauge mode $\omega_{\mathrm{G}}(\mathbf{q})$ and find, in particular, new Leggett modes $\omega_{\mathrm{L}}(\mathbf{q})$ with characteristic properties that are unique to non–centrosymmetric superconductors. We calculate their mass and dispersion that reflect the underlying spin–orbit coupling and thus the balance between triplet and singlet superconductivity occurring simultaneously. Finally, we demonstrate the role of the Anderson–Higgs mechanism: while the long–range Coulomb interaction shifts $\omega_{\mathrm{G}}(\mathbf{q})$ to the condensate plasma mode $\omega_{\mathrm{P}}(\mathbf{q})$, it leaves the mass Λ_0 of the new Leggett mode unaffected and only slightly modifies its dispersion.

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Introduction. Owing to the Pauli exclusion principle in single–band superconductors spin–singlet (even parity) and triplet (odd parity) pairing correlations never occur simultaneously. Important examples are spin–triplet odd–parity pairing correlations in superfluid ^{3}He [1, 2], triplet superconductivity in Sr_2RuO_4 [3], as well as unconventional singlet pairing correlations in heavy Fermion systems [4] and cuprates [5]. A necessary prerequisite for a clear singlet–triplet distinction is, however, the existence of an inversion center. The discovery of the bulk superconductors $CePt_3Si$ (tetragonal [6]) and $Li_2Pd_xPt_{3-x}B$ (cubic [7]), without inversion symmetry, to give only two examples, has therefore initiated extensive theoretical and experimental studies. The Rashba–type spin–orbit coupling caused by the absence of an inversion center implies (i) the lifting of the band degeneracy associated with a splitting into a two–band structure and (ii) the superposition of both singlet and triplet contributions to the superconducting gap [8, 9].

The breaking of a continuous symmetry in superconductors is associated with the occurrence of a gauge mode which is necessary to restore the charge conservation. Furthermore, in analogy to the Josephson effect, Leggett predicted the appearance of a new collective excitation in s–wave two–band superconductors, which corresponds to an out–of–phase oscillation mode of the phase difference of the coupled condensates [10]. So far, the Leggett mode has been only observed in MgB_2 [11], but several predictions for other s–wave superconductors have been made [12–14]. In non–centrosymmetric superconductors (NCS), however, where a complex mixing of singlet and triplet superconductivity occurs, it is not *a priori* clear whether a Leggett mode exists [9].

In this letter we use a microscopic theory to demonstrate the existence of Leggett modes in NCS. For this purpose, we calculate all order parameter collective modes associated with the condensate phase dynamics. For the first time we provide analytic expressions and numerical calculations for the gauge mode ω_{G}, the mass and the dispersion of Leggett's collective mode $\omega_{\mathrm{L}}(\mathbf{q})$, as well as for the frequency ω_{P} of the condensate plasma mode. The interesting interplay of these collective modes is studied in connection with the electromagnetic response of the pair condensate, with special emphasis on the participation of the collective modes in the Anderson–Higgs mechanism [15, 16]. We emphasize the calculation of the mass Λ_0 of various Leggett modes that depend strongly on the singlet–to–triplet ratio and may be observable by Raman or Brillouin scattering experiments.

Model description of NCS. The Hamiltonian for noninteracting electrons in a non–centrosymmetric crystal reads

$$\hat{\mathcal{H}} = \sum_{\mathbf{k}\sigma\sigma'} \hat{c}^{\dagger}_{\mathbf{k}\sigma} \left[\xi_{\mathbf{k}} \delta_{\sigma\sigma'} + \boldsymbol{\gamma}_{\mathbf{k}} \cdot \boldsymbol{\tau}_{\sigma\sigma'} \right] \hat{c}_{\mathbf{k}\sigma'} \ , \tag{1}$$

where $\xi_{\mathbf{k}}$ represents the bare band dispersion, $\sigma, \sigma' = \uparrow, \downarrow$ label the spin state and $\boldsymbol{\tau}$ are the Pauli matrices. The second term describes an antisymmetric spin–orbit coupling (ASOC) through the vector $\boldsymbol{\gamma}_{\mathbf{k}}$. In NCS two important classes of ASOCs are realized which reflect the underlying point group $\mathcal{G}$ of the crystal. We shall particularly be interested in the tetragonal point group C_{4v} (relevant for $CePt_3Si$) and the

cubic point group $O(432)$ (applicable to the system $Li_2Pd_xPt_{3-x}B$). For $\mathcal{G} = C_{4v}$ the ASOC reads

$$\boldsymbol{\gamma}_{\mathbf{k}} = \gamma_\perp(\hat{\mathbf{k}} \times \hat{\mathbf{e}}_z) + \gamma_\| \hat{k}_x\hat{k}_y\hat{k}_z(\hat{k}_x^2 - \hat{k}_y^2)\hat{\mathbf{e}}_z \ . \tag{2}$$

In the purely 2D case ($\gamma_\| = 0$) one recovers the Rashba interaction. For the cubic point group $\mathcal{G} = O(432)$ $\boldsymbol{\gamma}_{\mathbf{k}}$ reads $\boldsymbol{\gamma}_{\mathbf{k}} = \gamma_1\hat{\mathbf{k}} - \gamma_3[\hat{k}_x(\hat{k}_y^2 + \hat{k}_z^2)\hat{\mathbf{e}}_x + \hat{k}_y(\hat{k}_z^2 + \hat{k}_x^2)\hat{\mathbf{e}}_y + \hat{k}_z(\hat{k}_x^2 + \hat{k}_y^2)\hat{\mathbf{e}}_z]$.

What are the consequences of the ASOC? First, diagonalizing the Hamiltonian, one finds the energy eigenvalues $\xi_{\mathbf{k}\mu} = \xi_{\mathbf{k}} + \mu||\boldsymbol{\gamma}_{\mathbf{k}}||$ with $\mu = \pm 1$ which correspond to a lifting of the band degeneracy between the two spin states at a given momentum $\hbar\mathbf{k}$. This band splitting is responsible for the two–band structure characteristic of NCS metals. Second, the presence of an ASOC invalidates the classification of the superconducting order parameter with respect to spin singlet (even parity) and spin triplet (odd parity). Thus, in general, a linear combination of the gap on both bands is possible. Sigrist and co-workers have shown that most likely $\boldsymbol{\gamma}_{\mathbf{k}}$ orientates parallel to the $\mathbf{d}$–vector of the triplet part [8]. Thus, we can simply write the gap function on the two bands in terms of a singlet (Δ_s) and a triplet (Δ_{tr}) amplitude:

$$\Delta_{\mathbf{k}\mu} = \Delta_s(T) + \mu\Delta_{tr}(T) f_{\mathbf{k}} \ , \tag{3}$$

with $f_{\mathbf{k}} = ||\boldsymbol{\gamma}_{\mathbf{k}}||/[\langle||\boldsymbol{\gamma}_{\mathbf{k}'}||^2\rangle_{\mathrm{FS}}]^{1/2} \geq 0$, where $\langle\ldots\rangle_{\mathrm{FS}}$ denotes the Fermi surface (FS) average [17]. Thus, in short, while for all superconductors having an inversion center *either* singlet *or* triplet pairing is realized, in NCS singlet *and* triplet pairing occurs simultaneously. Simply speaking, the resulting ASOC may drive e.g. s– plus p–wave pairing on one band while s– minus p–wave is established on the other, leading to new collective modes.

Nonequilibrium Kinetic Theory for NCS. In order to calculate the dynamical properties of NCS we consider the response to a scalar electromagnetic potential $\phi(\mathbf{q}, \omega)$. In addition there contributes a charge fluctuation term, which accounts for the action of the 3D long–range Coulomb interaction $V_{\mathbf{q}} = 4\pi e^2/\mathbf{q}^2$ within the RPA, i.e. $\chi = \chi^{(0)}[1 - V_{\mathbf{q}}\chi^{(0)}]^{-1}$, where χ is a generalized response function. Then, the response to the perturbation $\delta\zeta \equiv e\phi(\mathbf{q}, \omega) + V_{\mathbf{q}}\delta n(\mathbf{q}, \omega)$ with δn being the total density response of the system, is described by a generalized momentum distribution function $\underline{n}^{\mu}_{\mathbf{p}\mathbf{p}'}$ which is a 2×2–matrix in Nambu–space using again the band basis of Eq. (3) with $\mu = \pm 1$. At the same time the perturbation $\delta\zeta$ induces fluctuations $\delta g^{(-)}_{\mathbf{k}\mu} \equiv \frac{1}{2}[\delta g_{\mathbf{k}\mu}\frac{\Delta^*_{\mathbf{k}\mu}}{|\Delta_{\mathbf{k}\mu}|} - \frac{\Delta_{\mathbf{k}\mu}}{|\Delta_{\mathbf{k}\mu}|}\delta g^*_{-\mathbf{k}\mu}]$ of the pairing amplitude $g_{\mathbf{k}\mu}$, as well as the important phase fluctuations of the superconducting order parameter $\delta\Delta^{(-)}_{\mathbf{k}\mu} \equiv \frac{1}{2}[\delta\Delta_{\mathbf{k}\mu}\frac{\Delta^*_{\mathbf{k}\mu}}{|\Delta_{\mathbf{k}\mu}|} - \frac{\Delta_{\mathbf{k}\mu}}{|\Delta_{\mathbf{k}\mu}|}\delta\Delta^*_{-\mathbf{k}\mu}]$, which we will later use to determine all collective modes. The Fourier transformation of $\underline{n}^{\mu}_{\mathbf{p}\mathbf{p}'}$ describes the evolution of the system in space and time after perturbation $\delta\zeta$. However, it is convenient to stay in $(\mathbf{q}, \omega)$–space and solve the von Neumann equation [18]

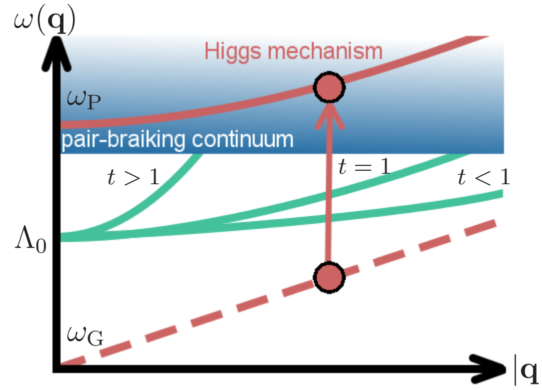

Figure 1. (color online) Illustration of various calculated collective modes (T=0) common to all NCS. The Anderson–Higgs mechanism shifts the gauge mode ω_{G} (dashed line) to the plasma mode ω_{P} usually lying in the pair–breaking continuum. The new Leggett modes (solid green lines) unique to NCS are only slightly changed by this process (not visible) and the mass Λ_0 remains unchanged. Importantly, in some cases $\Lambda_0 \to 0$ is possible, see discussion of Fig. 3; thus, the Leggett modes might be easy observable. Note that the slope of the Leggett modes depend on the ratio $t = \Delta_{tr}/\Delta_s$ as discussed in connection with Fig. 2.

$$\hbar\omega\, \underline{n}^{\mu}_{\mathbf{p}\mathbf{p}'} + \sum_{\mathbf{p}''}[\underline{n}^{\mu}_{\mathbf{p}\mathbf{p}''}, \underline{\xi}^{\mu}_{\mathbf{p}''\mathbf{p}'}] = 0 \tag{4}$$

in the clean limit, where $\mathbf{p} = \hbar\,(\mathbf{k} + \mathbf{q}/2)$, $\mathbf{p}' = \hbar\,(\mathbf{k} - \mathbf{q}/2)$ and the 2×2 energy matrix $\underline{\xi}^{\mu}_{\mathbf{p}''\mathbf{p}'}$ have been introduced. The simplest way to solve Eq. (4) is to make the following ansatz:

$$\begin{array}{lllll} \underline{n}^{\mu}_{\mathbf{p}\mathbf{p}'} & \equiv & \underline{n}_{\mathbf{k}\mu}(\mathbf{q}, \omega) & = & \underline{n}^0_{\mathbf{k}\mu}\delta_{\mathbf{q},0} + \delta\underline{n}_{\mathbf{k}\mu}(\mathbf{q}, \omega) \\ \underline{\xi}^{\mu}_{\mathbf{p}\mathbf{p}'} & \equiv & \underline{\xi}_{\mathbf{k}\mu}(\mathbf{q}, \omega) & = & \underline{\xi}^0_{\mathbf{k}\mu}\delta_{\mathbf{q},0} + \delta\underline{\xi}_{\mathbf{k}\mu}(\mathbf{q}, \omega) \end{array} \tag{5}$$

with the nonequilibrium quantities

$$\delta\underline{n}_{\mathbf{k}\mu} = \begin{pmatrix} \delta n_{\mathbf{k}\mu} & \mu\delta g_{\mathbf{k}\mu} \\ \mu\delta g^*_{\mathbf{k}\mu} & -\delta n_{-\mathbf{k}\mu} \end{pmatrix}, \delta\underline{\xi}_{\mathbf{k}\mu} = \begin{pmatrix} \delta\zeta & \mu\delta\Delta_{\mathbf{k}\mu} \\ \mu\delta\Delta^*_{\mathbf{k}\mu} & -\delta\zeta \end{pmatrix} .$$

After some lengthy, but straightforward calculations [supplement material, Eqs. (A.7)-(A.11)] we obtain from the off–diagonal components of Eq. (4) the rela-

tion between fluctuations of the pairing amplitude and fluctuations of the superconducting order parameter:

$$2\Delta_{\mathbf{k}\mu}[\delta g^{(-)}_{\mathbf{k}\mu}+\theta_{\mathbf{k}\mu}\delta\Delta^{(-)}_{\mathbf{k}\mu}] = \omega\lambda_{\mathbf{k}\mu}\delta\zeta-[\omega^2-(\mathbf{q}\cdot\mathbf{v}_{\mathbf{k}\mu})^2]\lambda_{\mathbf{k}\mu}\frac{\delta\Delta^{(-)}_{\mathbf{k}\mu}}{2\Delta_{\mathbf{k}\mu}}\ . \tag{6}$$

Here, we have identified the condensate response function

$$\lambda_{\mathbf{k}\mu}=4\Delta^2_{\mathbf{k}\mu}\frac{\theta_{\mathbf{k}\mu}[\omega^2-(\mathbf{q}\cdot\mathbf{v}_{\mathbf{k}\mu})^2]+\Phi_{\mathbf{k}\mu}(\mathbf{q}\cdot\mathbf{v}_{\mathbf{k}\mu})^2}{(\mathbf{q}\cdot\mathbf{v}_{\mathbf{k}\mu})^2[\omega^2-4\xi^2_{\mathbf{k}\mu}]-\omega^2[\omega^2-4E^2_{\mathbf{k}\mu}]} \tag{7}$$

with $\mathbf{v}_{\mathbf{k}\mu}=\partial\xi_{\mathbf{k}\mu}/\partial\hbar\mathbf{k}$, $\theta_{\mathbf{k}\mu}=\tanh(E_{\mathbf{k}\mu}/2k_{\mathrm{B}}T)/2E_{\mathbf{k}\mu}$, $E_{\mathbf{k}\mu}=[\xi^2_{\mathbf{k}\mu}+\Delta^2_{\mathbf{k}\mu}]^{1/2}$ and $\Phi_{\mathbf{k}\mu}=-\partial n_{\mathbf{k}\mu}/\partial\xi_{\mathbf{k}\mu}$ with momentum distribution function $n_{\mathbf{k}\mu}$. An important property of the condensate response is the sum rule, which generates the condensate density $\sum_{\mathbf{p}\mu}\lambda_{\mathbf{p}\mu}=N_0\sum_\mu\langle\lambda_{\hat{\mathbf{p}}\mu}\rangle_{\mathrm{FS}}\equiv N_0\lambda$, with $N_0=N_{\mathrm{F}}/2$ being the DoS for one spin projection. As we will show in supplement material [Eqs. (A.16)-(A.18)] the total particle density δn obeys the conservation law $\omega\delta n-\mathbf{q}\cdot\mathbf{j}=0$ only, if all phase fluctuation modes of the order parameter in Eq. (6) are properly accounted for.

Finally, we find from the diagonal components of Eq. (4) the density response of NCS:

$$\delta n_{\mathbf{k}\mu}=\left(\frac{(\mathbf{q}\cdot\mathbf{v}_{\mathbf{k}\mu})^2\,\varphi_{\mathbf{k}\mu}}{\omega^2-(\mathbf{q}\cdot\mathbf{v}_{\mathbf{k}\mu})^2}-\lambda_{\mathbf{k}\mu}\right)\delta\zeta+\omega\lambda_{\mathbf{k}\mu}\frac{\delta\Delta^{(-)}_{\mathbf{k}\mu}}{2\Delta_{\mathbf{k}\mu}} \tag{8}$$

with $\varphi_{\mathbf{k}\mu}=\Phi_{\mathbf{k}\mu}-\lambda_{\mathbf{k}\mu}$ being the quasiparticle response. Since we are only interested in the response of the superconducting condensate δn_{s}, we may ignore quasiparticle contributions $\propto\varphi_{\mathbf{k}\mu}$ in Eq. (8). Then, the density response function simplifies to $\delta n_{\mathbf{k}\mu}=-\lambda_{\mathbf{k}\mu}\delta\zeta+\omega\lambda_{\mathbf{k}\mu}\delta\Delta^{(-)}_{\mathbf{k}\mu}/2\Delta_{\mathbf{k}\mu}$. Hence, the condensate density response $\delta n_{\mathrm{s}}=\sum_{\mathbf{k}\mu}\delta n_{\mathbf{k}\mu}$ is exclusively determined by $\lambda_{\mathbf{k}\mu}$. In other words, we find that the frequency– and wave–vector dependence of $\delta n_{\mathrm{s}}(\mathbf{q},\omega)$ contains all information on the relevant order parameter collective modes in NCS. Finally, combining Eqs. (6) with both the superconducting gap equation $\Delta_{\mathbf{k}\mu}=\sum_{\mathbf{p}\nu}\Gamma^{\mu\nu}_{\mathbf{kp}}g_{\mathbf{p}\nu}$ and its variation

$$\delta\Delta^{(-)}_{\mathbf{k}\mu}=\sum_{\mathbf{p}\nu}\Gamma^{\mu\nu}_{\mathbf{kp}}\delta g^{(-)}_{\mathbf{p}\nu} \tag{9}$$

(with $\Gamma^{\mu\nu}_{\mathbf{kp}}$ being the pairing interaction [19]) leads to the main result of our analysis ($\hat{\mathbf{q}}=\mathbf{q}/|\mathbf{q}|$):

$$\delta n_{\mathrm{s}}(\mathbf{q},\omega)=N_0\lambda\frac{\omega^2_{\mathrm{G}}(\mathbf{q})[\omega^2-\omega'^2_{\mathrm{L}}(\mathbf{q})]}{\omega^4-[\omega^2_{\mathrm{P}}(\hat{\mathbf{q}})+\omega^2_{\mathrm{G}}(\mathbf{q})+\omega^2_{\mathrm{L}}(\mathbf{q})]\,\omega^2+[\omega^2_{\mathrm{P}}(\hat{\mathbf{q}})+\omega^2_{\mathrm{G}}(\mathbf{q})]\,\omega'^2_{\mathrm{L}}(\mathbf{q})}e\phi(\mathbf{q},\omega)\ . \tag{10}$$

New collective modes. From the denominator of Eq. (10) we can draw important conclusions which are summarized in Fig. 1. In analogy to neutral systems we first consider $\omega_{\mathrm{P}}(\hat{\mathbf{q}})\to 0$ and find two poles

$$\begin{aligned}\omega_1^2 &= \omega^2_{\mathrm{G}}(\mathbf{q})+\mathcal{O}\left(\tfrac{\omega^4_{\mathrm{G}}(\mathbf{q})}{\omega^2_{\mathrm{L}}(\mathbf{q})}\right) && \text{gauge mode}\\ \omega_2^2 &= \omega^2_{\mathrm{L}}(\mathbf{q})+\mathcal{O}\left(\tfrac{\omega^4_{\mathrm{G}}(\mathbf{q})}{\omega^2_{\mathrm{L}}(\mathbf{q})}\right) && \text{Leggett mode}\end{aligned} \tag{11}$$

with $\omega_{\mathrm{G}}(\mathbf{q})$ being the characteristic *gauge* mode of NCS with $\omega^2_{\mathrm{G}}(\mathbf{q})=\sum_\mu\left\langle\lambda_{\hat{\mathbf{p}}\mu}(\mathbf{q}\cdot\mathbf{v}_{\mathbf{p}\mu})^2\right\rangle_{\mathrm{FS}}/\lambda$. Furthermore, we discover the Anderson–Higgs mechanism for the gauge mode in NCS shifting it to the plasma frequency, i.e. $\omega^2_{\mathrm{P}}(\mathbf{q})=\omega^2_{\mathrm{P}}(\hat{\mathbf{q}})+\omega^2_{\mathrm{G}}(\mathbf{q})$. Thus, after Coulomb renormalization, we find:

$$\begin{aligned}\omega_1^2 &= \omega^2_{\mathrm{P}}(\hat{\mathbf{q}})+\omega^2_{\mathrm{G}}(\mathbf{q})+\mathcal{O}\left(\tfrac{\omega^2_{\mathrm{L}}(\mathbf{q})}{\omega^2_{\mathrm{P}}(\hat{\mathbf{q}})}\right) && \text{plasma mode}\\ \omega_2^2 &= \omega'^2_{\mathrm{L}}(\mathbf{q})+\mathcal{O}\left(\tfrac{\omega^2_{\mathrm{L}}(\mathbf{q})}{\omega^2_{\mathrm{P}}(\hat{\mathbf{q}})}\right) && \text{Leggett mode}\end{aligned} \tag{12}$$

with $\omega_{\mathrm{P}}(\hat{\mathbf{q}})$ being the characteristic *condensate plasma frequency* of NCS with $\omega^2_{\mathrm{P}}(\hat{\mathbf{q}})=4\pi ne^2\sum_\mu 3\left\langle\lambda_{\hat{\mathbf{p}}\mu}(\hat{\mathbf{q}}\cdot\hat{\mathbf{p}})^2\right\rangle_{\mathrm{FS}}/m$. It is important to note that the full condensate density response δn_{s} as described by Eq. (10) is also manifested in the condensate dielectric function $\epsilon\equiv 1-V_{\mathbf{q}}\delta n^{(0)}_{\mathrm{s}}/e\phi$, with $\delta n^{(0)}_{\mathrm{s}}\equiv\delta n_{\mathrm{s}}(\omega_{\mathrm{P}}(\hat{\mathbf{q}})\to 0)$. All in all, our new results for the gauge mode and plasma frequency generalizes the known solutions for ordinary two–band superconductors which can be obtained in the limit $f_{\mathbf{k}}\equiv 1$ [20].

The second pole in Eq. (10) leads with $\omega_{\mathrm{P}}(\hat{\mathbf{q}})\to 0$ to Eq. (11) determining the new *Leggett's collective modes* $\omega_{\mathrm{L}}(\mathbf{q})$ in NCS corresponding to oscillations in the relative phase of the superconducting condensates. The exact analytical result for $\omega^2_{\mathrm{L}}(\mathbf{q})$ is too lengthy to be shown here and thus can be found in the supplement material [see Eqs. (B3)-(B5)]. Instead, we illustrate its dispersion (for different $t=\Delta_{tr}/\Delta_s$) in Fig. 1 and calculate its slope (as an example for C_{4v}) in Fig. 2. As expected, we find for all point groups considered the dispersion $\left(\omega^2_{\mathrm{L}}(\mathbf{q})-\Lambda^2_0\right)\propto|\mathbf{q}|^2$. The slope, however, depends on the ratio $t=\Delta_{tr}/\Delta_s$. Thus, in Fig. 2(a) we show the slope of the Leggett mode exemplarily for the tetragonal point group C_{4v} [see Eq. (2)] along the $\hat{\mathbf{q}}_x$– and $\hat{\mathbf{q}}_y$–direction. The calculated up-

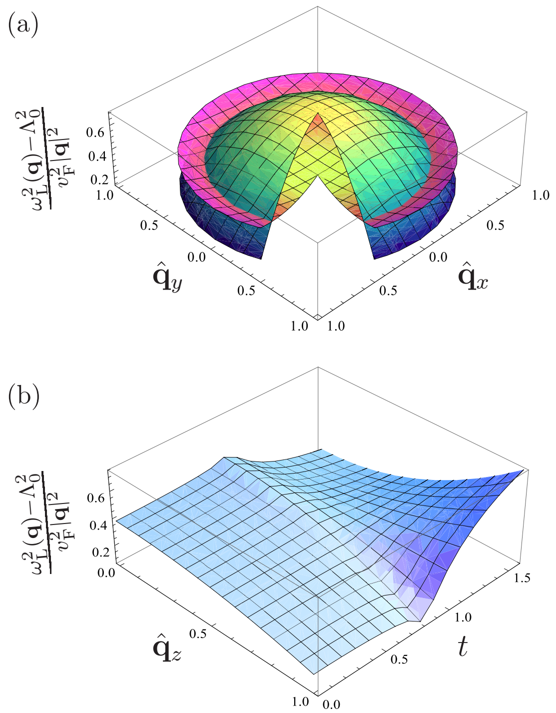

Figure 2. (color online) Slope of the dispersion of the Leggett mode for NCS systems with C_{4v} point group symmetry as a function of the unit vectors $\hat{\mathbf{q}}_x, \hat{\mathbf{q}}_y, \hat{\mathbf{q}}_z$. (a) Comparison of the slope for $t = 0.5$ (upward parabola) and $t = 1.5$ (downward parabola), (b) slope along the $\hat{\mathbf{q}}_z$–direction for various $t = \Delta_{tr}/\Delta_s$.

ward parabola corresponds to $t = 0.5$ while the downward parabola corresponds to $t = 1.5$, respectively. For $t = 1$ one finds a constant slope of 1/3 (independent of $\hat{\mathbf{q}}_x$ and $\hat{\mathbf{q}}_y$, not shown). The three resulting dispersions are illustrated schematically in Fig. 1. In Fig. 2(b) we show the slope along the $\hat{\mathbf{q}}_z$–direction for various t which reveals a non–monotonic behavior for fixed $\hat{\mathbf{q}}_z$. In contrast, for the cubic point group $O(432)$ we find in all directions $\omega_L^2(\mathbf{q}) - \Lambda_0^2 = \frac{1}{3}\mathbf{v}_F^2|\mathbf{q}|^2$ *independent* of t (not shown), since the underlying ASOC is isotropic to leading order. This would correspond to the curve with $t = 1$ in Fig. 1.

From $\omega_2^2(\mathbf{q})$ in Eq. (11), we find the mass Λ_0 of the Leggett mode

$$\Lambda_0^2 \equiv \omega_L^2(\mathbf{q}=0) = 4\gamma_{\rm ncs}\Delta_s\Delta_{tr}\frac{\lambda}{\lambda_0\lambda_2-\lambda_1^2}\;, \qquad (13)$$

where the definitions

$$\lambda_n = \Delta_s\Delta_{tr}\sum_{\mu=\pm1}\left\langle\lambda_{\hat{\mathbf{p}}\mu}(\mu f_{\mathbf{p}})^n/\Delta_{\mathbf{p}\mu}^2\right\rangle_{\rm FS} \qquad (14)$$

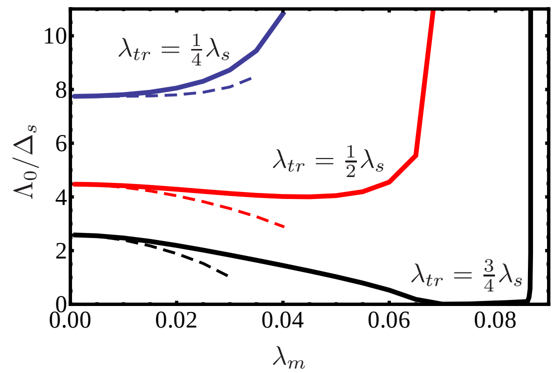

Figure 3. (color online) Normalized mass of the Leggett mode for NCS systems with C_{4v} point group symmetry for fixed $\lambda_s = 0.1$ as a function of the mixing term and various λ_{tr}: $\lambda_{tr} = 0.025$ (upper solid line), $\lambda_{tr} = 0.05$ (middle solid line) and $\lambda_{tr} = 0.075$ (lower solid line). The dashed lines correspond to Eq. (18) which is an analytical solution in the limit of small t.

have been used. Here, γ_{ncs} represents the coupling strength of the Leggett mode, which we will calculate below. In order to determine Λ_0 we need the *exact* solution of the coupled self–consistency equations of the superconducting gap functions [see also Eq. (3)]: $\Delta_{\mathbf{k}\mu} = \sum_{\mathbf{p}\nu=\pm1}\Gamma_{\mathbf{kp}}^{\mu\nu}g_{\mathbf{p}\nu}$ with $g_{\mathbf{p}\nu} = -\theta_{\mathbf{p}\nu}\Delta_{\mathbf{p}\nu}$ being the pairing amplitude and $\theta_{\mathbf{p}\nu}$ has been defined together with Eq. (7). We choose the generalized two–gap weak–coupling pairing interaction of Ref. [21] $\Gamma_{\mathbf{kp}}^{\mu\nu} = -\left\{\Gamma_s + \Gamma_{tr}\mu\nu f_{\mathbf{k}}f_{\mathbf{p}} + \Gamma_m\left(\mu f_{\mathbf{k}} + \nu f_{\mathbf{p}}\right)\right\}$ $\Theta(\epsilon_0 - |\xi_{\mathbf{k}\mu}|)\Theta(\epsilon_0 - |\xi_{\mathbf{p}\nu}|)$ and obtain

$$\left\{-\boldsymbol{\lambda}^{-1} + \begin{pmatrix}\Xi_0 & \Xi_1\\ \Xi_1 & \Xi_2\end{pmatrix}\right\}\cdot\begin{pmatrix}\Delta_s\\ \Delta_{tr}\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix} \qquad (15)$$

with $\lambda_\alpha = N(0)\Gamma_\alpha$, $\alpha = s, tr, m$,

$$\boldsymbol{\lambda} = \begin{pmatrix}\lambda_s & \lambda_m\\ \lambda_m & \lambda_{tr}\end{pmatrix}\;;\;\boldsymbol{\lambda}^{-1} = \frac{1}{|\boldsymbol{\lambda}|}\begin{pmatrix}\lambda_{tr} & -\lambda_m\\ -\lambda_m & \lambda_s\end{pmatrix} \quad (16)$$

and $\Xi_n = \sum_\mu\langle\theta_{\hat{\mathbf{p}}\mu}\left(\mu f_{\mathbf{p}}\right)^n\rangle_{\rm FS}$. Note that one obtains the ordinary two–band case if $\Xi_1 \to 0$ [22]. Equations (15)–(16) have the advantage that the exact relation

$$\gamma_{\rm ncs} = \lambda_m/|\boldsymbol{\lambda}| + \Xi_1 \qquad (17)$$

holds and thus determines the coupling constant in Eq. (13). Thus, for given $\lambda_s, \lambda_{tr}, \lambda_m$ a numerical exact solution of Eq. (15) is always possible: the resulting exact gap function $\Delta_{\mathbf{k}\mu}$ needs to be inserted in Eqs. (13) and (14) to determine Λ_0 [23].

In Fig. 3 we show results for the Leggett mass Λ_0 for fixed $\lambda_s = 0.1$ as a function of λ_m. While for a

small triplet contribution (upper solid line) Λ_0 increases monotonically, we find a non–monotonic behavior of the mass for increasing λ_{tr} (middle solid line). Finally, if $\lambda_s \approx \lambda_{tr}$ we obtain the important case that Λ_0 can become zero (lower solid line). Physically, this corresponds to a partly vanishing gap on one of the Fermi surfaces [see Eq. (3) and Ref. [24]]. Also displayed in Fig. 3 is the analytical solution in the limit of small t (dashed lines)

$$\Lambda_0^2 = 2\Delta_s^2 \frac{\lambda_s - \lambda_{tr}}{|\boldsymbol{\lambda}|} \left[1 - \left(3 \left\langle f_{\mathbf{k}}^4 \right\rangle_{\rm FS} - 1\right) t^2\right] \ . \quad (18)$$

This might help experimentalists to estimate in which materials the new Leggett modes are most easiest observable.

Finally, we return to the Anderson–Higgs mechanism. What is its role for the new Leggett modes? First, we conclude that the Leggett mass Λ_0 is unchanged, since the r.h.s. of Eq. (13) does not depend on $\omega_{\rm G}$. Physically, this corresponds to the fact that the Meissner effect in the presence of a new Leggett mode is unchanged. Second, we find that the dispersion of the Leggett mode is only slightly changed. To see this, one needs to consider the difference in ω_2^2 between Eqs. (11) and (12). Since $\omega_{\rm G}(\mathbf{q} \to 0) \to 0$ and $\omega_{\rm P} \gg \omega_{\rm L}$, the higher order corrections nearly vanish. The resulting $(\omega_{\rm L}^2 - \omega_{\rm L}'^2)$ is also very small [see supplement material Eq. (C.4)]. Thus, we conclude that the dispersion of the Leggett mode and the results shown in Fig. 2 are nearly unchanged due to the Anderson–Higgs mechanism [25].

In conclusion, using a gauge–invariant theory of superconducting phase fluctuations in NCS we have demonstrated the existence of Leggett modes and calculated their characteristic mass and dispersion for various crystal symmetries. Both properties reflect the underlying spin–orbit coupling and depend strongly on the singlet–to–triplet ratio. Furthermore, we have calculated the corresponding gauge modes and clarified the role of the Anderson–Higgs mechanism for collective modes in NCS.

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Leggett modes and the Anderson–Higgs mechanism in superconductors without inversion symmetry Supplement material

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A. KINETIC THEORY FOR NCS.

Model description in equilibrium. A non–centrosymmetric superconductor (NCS) is described in equilibrium by the Hamiltonian $\hat{\mathcal{H}}$, which is given by Eq. (1) in the main text. In order to include the pairing correlations into the description, we extend Eq. (1) to include the gap matrix $\mathbf{\Delta_k}$ as an off-diagonal element of an energy matrix $\underline{\xi}^0_{\mathbf{k}}$ in Nambu space. In the presence of an antisymmetric spin–orbit coupling (ASOC), represented by the vector $\boldsymbol{\gamma}_{\mathbf{k}}$, the 4×4 energy matrix has the following form in the spin representation:

$$\underline{\xi}^0_{\mathbf{k}} = \begin{pmatrix} \xi_{\mathbf{k}}\mathbf{1} + \boldsymbol{\gamma}_{\mathbf{k}}\cdot\boldsymbol{\tau} & \mathbf{\Delta_k} \\ \mathbf{\Delta}^\dagger_{\mathbf{k}} & -[\xi_{-\mathbf{k}}\mathbf{1} + \boldsymbol{\gamma}_{-\mathbf{k}}\cdot\boldsymbol{\tau}]^T \end{pmatrix} \tag{A.1}$$

In order to account for the two–band structure occurring in NCS systems in the limit of large spin–orbit coupling, it is convenient to perform a unitary transformation of $\underline{\xi}^0_{\mathbf{k}}$ into the *helicity–band basis* or simply *band basis*. The transformation from spin to band basis is described by the matrix $\mathbf{U_k}$, which has the property

$$\mathbf{U}^\dagger_{\mathbf{k}}(\boldsymbol{\gamma}_{\mathbf{k}}\cdot\boldsymbol{\tau})\mathbf{U_k} = \|\boldsymbol{\gamma}_{\mathbf{k}}\|\,\boldsymbol{\tau}^3 \tag{A.2}$$

and which is obtained in the form of a SU(2) rotation

$$\mathbf{U_k} = \mathrm{e}^{-i\frac{\theta_\gamma}{2}\hat{\mathbf{n}}_\gamma\cdot\boldsymbol{\tau}}\ ;\ \cos\theta_\gamma = \hat{\boldsymbol{\gamma}}_{\mathbf{k}}\cdot\hat{\mathbf{z}}\ ;\ \mathbf{n}_\gamma = \frac{\boldsymbol{\gamma}_{\mathbf{k}}\times\hat{\mathbf{z}}}{\|\boldsymbol{\gamma}_{\mathbf{k}}\times\hat{\mathbf{z}}\|} \tag{A.3}$$

that corresponds to a rotation in spin space into the $\hat{\mathbf{z}}$–direction about the polar angle θ_γ between $\boldsymbol{\gamma}_{\mathbf{k}}$ and $\hat{\mathbf{z}}$. Here, $\boldsymbol{\tau}$ denotes the vector of Pauli spin matrices. A straightforward extension of this transformation into Nambu space reads [S.1]

$$\underline{\xi}^{(band)}_{\mathbf{k}} \equiv \underline{U}^\dagger_{\mathbf{k}}\underline{\xi}^0_{\mathbf{k}}\underline{U}_{\mathbf{k}} = \begin{pmatrix} \xi_{\mathbf{k}+} & 0 & 0 & \Delta_{\mathbf{k}+} \\ 0 & \xi_{\mathbf{k}-} & -\Delta_{\mathbf{k}-} & 0 \\ 0 & -\Delta^*_{\mathbf{k}-} & -\xi_{\mathbf{k}-} & 0 \\ \Delta^*_{\mathbf{k}+} & 0 & 0 & -\xi_{\mathbf{k}+} \end{pmatrix}\ ;\ \underline{U}_{\mathbf{k}} = \begin{pmatrix} \mathbf{U_k} & 0 \\ 0 & \mathbf{U}^*_{\mathbf{k}} \end{pmatrix} \tag{A.4}$$

with the energy values $\xi_{\mathbf{k}\mu} = \xi_{\mathbf{k}} + \mu||\boldsymbol{\gamma}_{\mathbf{k}}||$ and the gap functions $\Delta_{\mathbf{k}\mu} = \Delta_s(T) + \mu\Delta_{tr}(T)f_{\mathbf{k}}$ also given by Eq. (3) in the main text. Introducing a band–index $\mu = \pm 1$, one may write the equilibrium energy matrix in the band basis in the compact form:

$$\underline{\xi}^0_{\mathbf{k}\mu} = \begin{pmatrix} \xi_{\mathbf{k}\mu} & \mu\Delta_{\mathbf{k}\mu} \\ \mu\Delta^*_{\mathbf{k}\mu} & -\xi_{-\mathbf{k}\mu} \end{pmatrix} \tag{A.5}$$

In analogy, one can find for the equilibrium density matrix:

$$\underline{n}^0_{\mathbf{k}\mu} = \begin{pmatrix} \frac{1}{2} - \xi_{\mathbf{k}\mu}\theta_{\mathbf{k}\mu} & -\mu\Delta_{\mathbf{k}\mu}\theta_{\mathbf{k}\mu} \\ -\mu\Delta^*_{\mathbf{k}\mu}\theta_{\mathbf{k}\mu} & \frac{1}{2} + \xi_{\mathbf{k}\mu}\theta_{\mathbf{k}\mu} \end{pmatrix} \tag{A.6}$$

Nonequilibrium Kinetic Equations. The action of an external perturbation $\delta\zeta = e\phi(\mathbf{q},\omega) + V_{\mathbf{q}}\delta n(\mathbf{q},\omega)$ leads to the deviation of the density matrix, as well as the energy matrix, from its equilibrium value. An NCS is now described in the band basis by a generalized momentum distribution function $\underline{n}^\mu_{\mathbf{pp'}}$ and an energy matrix

$\underline{\xi}^{\mu}_{\mathbf{pp}'}$, respectively. A collisionless quantum dynamics is given by the von Neumann equation [see Eq. (4) in the main text]:

$$\hbar\omega\,\underline{n}^{\mu}_{\mathbf{pp}'} + \sum_{\mathbf{p}''}[\underline{n}^{\mu}_{\mathbf{pp}''}, \underline{\xi}^{\mu}_{\mathbf{p}''\mathbf{p}'}] = 0 \tag{A.7}$$

This equation can be linearized by using the ansatz of Eq. (5) given in the main text. This leads to

$$\hbar\omega\delta\underline{n}_{\mathbf{k}\mu} + \delta\underline{n}_{\mathbf{k}\mu}\underline{\xi}^{0}_{\mathbf{k}-\frac{\mathbf{q}}{2}\mu} - \underline{\xi}^{0}_{\mathbf{k}+\frac{\mathbf{q}}{2}\mu}\delta\underline{n}_{\mathbf{k}\mu} = \delta\underline{\xi}_{\mathbf{k}\mu}\underline{n}^{0}_{\mathbf{k}-\frac{\mathbf{q}}{2}\mu} - \underline{n}^{0}_{\mathbf{k}+\frac{\mathbf{q}}{2}\mu}\delta\underline{\xi}_{\mathbf{k}\mu} \tag{A.8}$$

with the equilibrium quasiparticle energy $\underline{\xi}^{0}_{\mathbf{k}\mu}$ and the distribution function $\underline{n}^{0}_{\mathbf{k}\mu}$ defined in Eqs. (A.5) and (A.6), respectively. The momentum and frequency–dependent deviation from equilibrium can be defined in the appropriate way as 2×2 matrices in the Nambu space:

$$\delta\underline{n}_{\mathbf{k}\mu} = \begin{pmatrix} \delta n_{\mathbf{k}\mu} & \mu\delta g_{\mathbf{k}\mu} \\ \mu\delta g^{*}_{\mathbf{k}\mu} & -\delta n_{-\mathbf{k}\mu} \end{pmatrix} \quad \text{and} \quad \delta\underline{\xi}_{\mathbf{k}\mu} = \begin{pmatrix} \delta\xi_{\mathbf{k}\mu} & \mu\delta\Delta_{\mathbf{k}\mu} \\ \mu\delta\Delta^{*}_{\mathbf{k}\mu} & -\delta\xi_{-\mathbf{k}\mu} \end{pmatrix} \tag{A.9}$$

with $\delta\xi_{\mathbf{k}\mu} = \delta\xi_{-\mathbf{k}\mu} = \delta\zeta$. Thus, the equation (A.8) represents a set of eight equations in the band basis [S.2] (with the band index $\mu = \pm 1$). Furthermore, it is convenient to decompose the diagonal elements of the energy and density deviation matrices according to their parity with respect to $\mathbf{k} \to -\mathbf{k}$

$$\begin{aligned} \delta n^{(s)}_{\mathbf{k}\mu} &= \frac{1}{2}\left(\delta n_{\mathbf{k}\mu} + s\delta n_{-\mathbf{k}\mu}\right) \\ \delta\xi^{(s)}_{\mathbf{k}\mu} &= \frac{1}{2}\left(\delta\xi_{\mathbf{k}\mu} + s\delta\xi_{-\mathbf{k}\mu}\right) \end{aligned} \tag{A.10}$$

with the labeling $s = \pm 1$. By analogy, the off–diagonal components are decomposed into their real and imaginary parts:

$$\begin{aligned} \delta g^{(s)}_{\mathbf{k}\mu} &= \frac{1}{2}\left(\delta g_{\mathbf{k}\mu}\frac{\Delta^{*}_{\mathbf{k}\mu}}{|\Delta_{\mathbf{k}\mu}|} + s\frac{\Delta_{\mathbf{k}\mu}}{|\Delta_{\mathbf{k}\mu}|}\delta g^{*}_{-\mathbf{k}\mu}\right) \\ \delta\Delta^{(s)}_{\mathbf{k}\mu} &= \frac{1}{2}\left(\delta\Delta_{\mathbf{k}\mu}\frac{\Delta^{*}_{\mathbf{k}\mu}}{|\Delta_{\mathbf{k}\mu}|} + s\frac{\Delta_{\mathbf{k}\mu}}{|\Delta_{\mathbf{k}\mu}|}\delta\Delta^{*}_{-\mathbf{k}\mu}\right) \end{aligned} \tag{A.11}$$

where $\delta\Delta^{(+)}_{\mathbf{k}\mu}$ represents the amplitude fluctuations and $\Delta^{(-)}_{\mathbf{k}\mu}$ the phase fluctuations of the order parameter. After these specifications the off–diagonal components of the Eq. (A.8) simplify to [S.2]:

$$\delta g^{(+)}_{\mathbf{k}\mu} = -\left(\theta_{\mathbf{k}\mu} + \frac{\omega^2 - (\mathbf{v}_{\mathbf{k}\mu}\cdot\mathbf{q})^2 - 4\Delta^2_{\mathbf{k}\mu}}{4\Delta^2_{\mathbf{k}\mu}}\lambda_{\mathbf{k}\mu}\right)\delta\Delta^{(+)}_{\mathbf{k}\mu} \tag{A.12}$$

$$\delta g^{(-)}_{\mathbf{k}\mu} + \theta_{\mathbf{k}\mu}\delta\Delta^{(-)}_{\mathbf{k}\mu} = \frac{\omega\lambda_{\mathbf{k}\mu}}{2\Delta_{\mathbf{k}\mu}}\delta\zeta - [\omega^2 - (\mathbf{q}\cdot\mathbf{v}_{\mathbf{k}\mu})^2]\lambda_{\mathbf{k}\mu}\frac{\delta\Delta^{(-)}_{\mathbf{k}\mu}}{4\Delta^2_{\mathbf{k}\mu}} \tag{A.13}$$

whereas for the diagonal elements one gets:

$$\delta n^{(+)}_{\mathbf{k}\mu} = \left(\frac{(\mathbf{q}\cdot\mathbf{v}_{\mathbf{k}\mu})^2\varphi_{\mathbf{k}\mu}}{\omega^2 - (\mathbf{q}\cdot\mathbf{v}_{\mathbf{k}\mu})^2} - \lambda_{\mathbf{k}\mu}\right)\delta\zeta + \omega\lambda_{\mathbf{k}\mu}\frac{\delta\Delta^{(-)}_{\mathbf{k}\mu}}{2\Delta_{\mathbf{k}\mu}} \tag{A.14}$$

$$\delta n^{(-)}_{\mathbf{k}\mu} = \frac{\omega\,(\mathbf{q}\cdot\mathbf{v}_{\mathbf{k}\mu})\,\varphi_{\mathbf{k}\mu}}{\omega^2 - (\mathbf{q}\cdot\mathbf{v}_{\mathbf{k}\mu})^2}\delta\zeta + (\mathbf{v}_{\mathbf{k}\mu}\cdot\mathbf{q})\,\lambda_{\mathbf{k}\mu}\frac{\delta\Delta^{(-)}_{\mathbf{k}\mu}}{2\Delta_{\mathbf{k}\mu}} \tag{A.15}$$

Equation (A.13) describes the important relation between fluctuation of the pairing amplitude and the phase fluctuations of the superconducting order parameter [see Eq. (6) in the main text]. The density response to a

scalar perturbation $\delta\zeta$ is given by Eq. (A.14) [corresponding to Eq. (8) in the main text with $\delta n_{\mathbf{k}\mu} \equiv \delta n^{(+)}_{\mathbf{k}\mu}$]

Conservation law. One strength of the matrix kinetic equation approach lies in the straightforward physical interpretation of its results. In addition, the gauge invariance of the whole theory can be demonstrated easily if *all* phase fluctuation modes of the order parameter are properly taken into account: As one can see from Eqs. (A.14)-(A.15) the density distribution functions $\delta n^{(s)}_{\mathbf{k}\mu}$ are directly connected with the phase fluctuations of the order parameter $\delta\Delta^{(-)}_{\mathbf{k}\mu}$. The combination of the results from Eqs. (A.14)- (A.15) yields together with the subsequent integration over the momentum space $\mathbf{k}$ to the continuity equation

$$\omega\delta n - \mathbf{q}\cdot\mathbf{j} = \sum_{\mathbf{p}\mu} \lambda_{\mathbf{p}\mu} \left\{ \left[\omega^2 - (\mathbf{q}\cdot\mathbf{v}_{\mathbf{p}\mu})^2\right] \frac{\delta\Delta^{(-)}_{\mathbf{p}\mu}}{2\Delta_{\mathbf{p}\mu}} - \omega\delta\zeta \right\} \tag{A.16}$$

which at first glance displays a non–vanishing right-hand side. However, by using Eqs. (A.12)- (A.13) and the variation of the energy gap equation

$$\delta\Delta^{(-)}_{\mathbf{k}\mu} = \sum_{\mathbf{p}\nu} \Gamma^{\mu\nu}_{\mathbf{kp}} \delta g^{(-)}_{\mathbf{p}\nu} \tag{A.17}$$

one finds after a straightforward, but lengthy calculation:

$$\omega\delta n - \mathbf{q}\cdot\mathbf{j} = 0\ . \tag{A.18}$$

Thus, the particle conservation and, associated with it, the gauge invariance of the theory are satisfied within the framework of the matrix kinetic theory.

B. NEW COLLECTIVE MODES

The collective excitations of a non–centrosymmetric system can be obtained from the condition, that the denominator of Eq. (10) vanishes, i.e.

$$\omega^4 - \left[\omega^2_{\mathrm{P}}(\hat{\mathbf{q}}) + \omega^2_{\mathrm{G}}(\mathbf{q}) + \omega^2_{\mathrm{L}}(\mathbf{q})\right]\omega^2 + \left[\omega^2_{\mathrm{P}}(\hat{\mathbf{q}}) + \omega^2_{\mathrm{G}}(\mathbf{q})\right]\omega'^2_{\mathrm{L}}(\mathbf{q}) = 0 \tag{B.1}$$

with $\omega^2_{\mathrm{G}}(\mathbf{q}) = \sum_\mu \left\langle \lambda_{\hat{\mathbf{p}}\mu}(\mathbf{q}\cdot\mathbf{v}_{\mathbf{p}\mu})^2\right\rangle_{\mathrm{FS}}/\lambda$ and $\omega^2_{\mathrm{P}}(\hat{\mathbf{q}}) = 4\pi n e^2 \sum_\mu 3\left\langle \lambda_{\hat{\mathbf{p}}\mu}(\hat{\mathbf{q}}\cdot\hat{\mathbf{p}})^2\right\rangle_{\mathrm{FS}}/m$. Here, we also use the abbreviation

$$\omega'^2_{\mathrm{L}}(\mathbf{q}) = \Lambda^2_0 + \frac{\alpha_0\alpha_2\omega^2_{\mathbf{q}0}\omega^2_{\mathbf{q}2} - \alpha^2_1\omega^4_{\mathbf{q}1}}{(\alpha_0\alpha_2 - \alpha^2_1)\,\omega^2_{\mathrm{G}}(\mathbf{q})} \tag{B.2}$$

and

$$\omega^2_{\mathrm{L}}(\mathbf{q}) = \Lambda^2_0 + \frac{\alpha_0\alpha_2\left(\omega^2_{\mathbf{q}0} + \omega^2_{\mathbf{q}2}\right) - 2\alpha^2_1\omega^2_{\mathbf{q}1}}{(\alpha_0\alpha_2 - \alpha^2_1)} - \omega^2_{\mathrm{G}}(\mathbf{q}) \tag{B.3}$$

with the quantities α_n, which are defined as

$$\alpha_n = \sum_\mu \left\langle \frac{\lambda_{\mathbf{k}\mu}}{\Delta^2_{\mathbf{k}\mu}} (\mu f_{\mathbf{k}})^n \right\rangle_{\mathrm{FS}} \tag{B.4}$$

together with:

$$\omega^2_{\mathbf{q}n} = \frac{1}{\alpha_n} \sum_\mu \left\langle \frac{\lambda_{\mathbf{k}\mu}}{\Delta^2_{\mathbf{k}\mu}} (\mathbf{q}\cdot\mathbf{v}_{\mathbf{k}\mu})^2 (\mu f_{\mathbf{k}})^n \right\rangle_{\mathrm{FS}} \tag{B.5}$$

and the Leggett mass $\Lambda_0^2 \equiv \omega_{\rm L}^2(\mathbf{q}=0)$. Thus, Eq. (B.1) is a quadratic equation with respect to ω^2 with the solutions

$$\omega_{1,2}^2 = \frac{1}{2}\left[\omega_{\rm P}^2(\hat{\mathbf{q}}) + \omega_{\rm G}^2(\mathbf{q}) + \omega_{\rm L}^2(\mathbf{q}) \pm \left(\omega_{\rm P}^2(\hat{\mathbf{q}}) + \omega_{\rm G}^2(\mathbf{q}) + \omega_{\rm L}^2(\mathbf{q})\right)\sqrt{1 - 4\frac{\left[\omega_{\rm P}^2(\hat{\mathbf{q}}) + \omega_{\rm G}^2(\mathbf{q})\right]\omega_{\rm L}'^2(\mathbf{q})}{\left(\omega_{\rm P}^2(\hat{\mathbf{q}}) + \omega_{\rm G}^2(\mathbf{q}) + \omega_{\rm L}^2(\mathbf{q})\right)^2}}\,\right]. \quad \text{(B.6)}$$

This result can be further simplified by using a Taylor expansion of the square root. Therefore, by considering terms up to second order in $|\mathbf{q}|$ one gets:

$$\omega_{1,2}^2 = \omega_{\rm P}^2(\hat{\mathbf{q}}) + \omega_{\rm G}^2(\mathbf{q}) + \omega_{\rm L}^2(\mathbf{q}) \pm \frac{\left[\omega_{\rm P}^2(\hat{\mathbf{q}}) + \omega_{\rm G}^2(\mathbf{q})\right]\omega_{\rm L}'^2(\mathbf{q})}{\omega_{\rm P}^2(\hat{\mathbf{q}}) + \omega_{\rm G}^2(\mathbf{q}) + \omega_{\rm L}^2(\mathbf{q})} \quad \text{(B.7)}$$

In the absence of the long–range Coulomb interaction (i.e. for the case $\omega_{\rm P}^2(\hat{\mathbf{q}}) = 0$) one finds from Eq. (B.7) following result for the collective modes [see Eq. (11) in the main text]:

$$\begin{aligned} \omega_1^2 &= \omega_{\rm G}^2(\mathbf{q}) + \mathcal{O}\left(\frac{\omega_{\rm G}^4(\mathbf{q})}{\omega_{\rm L}^2(\mathbf{q})}\right) \qquad \text{Gauge mode} \\ \omega_2^2 &= \omega_{\rm L}^2(\mathbf{q}) + \mathcal{O}\left(\frac{\omega_{\rm G}^4(\mathbf{q})}{\omega_{\rm L}^2(\mathbf{q})}\right) \qquad \text{Leggett mode} \end{aligned} \quad \text{(B.8)}$$

The Coulomb interaction leads to the renormalization of this result [see Eq. (12) in the main text]:

$$\begin{aligned} \omega_1^2 &= \omega_{\rm P}^2(\hat{\mathbf{q}}) + \omega_{\rm G}^2(\mathbf{q}) + \omega_{\rm L}^2(\mathbf{q}) - \omega_{\rm L}'^2(\mathbf{q}) + \mathcal{O}\left(\frac{\omega_{\rm L}^2(\mathbf{q})}{\omega_{\rm P}^2(\hat{\mathbf{q}})}\right) \qquad \text{Plasma mode} \\ \omega_2^2 &= \omega_{\rm L}'^2(\mathbf{q}) + \mathcal{O}\left(\frac{\omega_{\rm L}^2(\mathbf{q})}{\omega_{\rm P}^2(\hat{\mathbf{q}})}\right) \qquad \text{Leggett mode} \end{aligned} \quad \text{(B.9)}$$

Thus, the mass of the Leggett mode remains unaffected by this process, but its dispersion is changed. In the limiting case of small $\mathbf{q}$ the dispersion modification is, however, negligible.

C. ANDERSON–HIGGS MECHANISM

In order to discuss the Anderson–Higgs mechanism for the Leggett mode in non–centrosymmetric superconductors we consider the difference between the Coulomb–renormalized Leggett mode $\omega_{\rm L}'$ defined in Eq. (B.2) and its unrenormalized counterpart $\omega_{\rm L}$ defined in Eq. (B.3):

$$\omega_{\rm L}^2(\mathbf{q}) - \omega_{\rm L}'^2(\mathbf{q}) = \frac{\alpha_0\alpha_2\left(\omega_{\mathbf{q}0}^2 + \omega_{\mathbf{q}2}^2\right) - 2\alpha_1^2\omega_{\mathbf{q}1}^2}{\left(\alpha_0\alpha_2 - \alpha_1^2\right)} - \omega_{\rm G}^2(\mathbf{q}) - \frac{\alpha_0\alpha_2\omega_{\mathbf{q}0}^2\omega_{\mathbf{q}2}^2 - \alpha_1^2\omega_{\mathbf{q}1}^4}{\left(\alpha_0\alpha_2 - \alpha_1^2\right)\omega_{\rm G}^2(\mathbf{q})} \quad \text{(C.1)}$$

For simplicity we make following assumptions: (i) low temperature limit ($T \to 0$); (ii) isotropic spin–orbit coupling $f_{\mathbf{k}} = 1$ [corresponding to the leading order of $\gamma_{\mathbf{k}}$ for the cubic point group $O(432)$]. A generalization beyond these approximations is, however, straightforward. With these assumptions, equation (C.1) simplifies to

$$\omega_{\rm L}^2(\mathbf{q}) - \omega_{\rm L}'^2(\mathbf{q}) = \frac{2\alpha_0^2\omega_{\mathbf{q}0}^2 - 2\alpha_1^2\omega_{\mathbf{q}1}^2}{\left(\alpha_0^2 - \alpha_1^2\right)} - \omega_{\rm G}^2(\mathbf{q}) - \frac{\alpha_0^2\omega_{\mathbf{q}0}^4 - \alpha_1^2\omega_{\mathbf{q}1}^4}{\left(\alpha_0^2 - \alpha_1^2\right)\omega_{\rm G}^2(\mathbf{q})} \quad \text{(C.2)}$$

with $\omega_{\rm G}^2 = \frac{1}{6}\sum_\mu v_{{\rm F}\mu}^2|\mathbf{q}|^2$ and $v_{{\rm F}\mu}$ being the Fermi velocity on the band $\mu = \pm 1$. By using the definitions (B.4) and (B.5) the equation (C.2) can be further simplified. Thus, after straightforward calculations one obtains:

$$\omega_{\rm L}^2(\mathbf{q}) - \omega_{\rm L}'^2(\mathbf{q}) = \omega_{\rm G}^2(\mathbf{q}) - \frac{\left\langle\left(\mathbf{q}\cdot\mathbf{v}_{\mathbf{k}+}\right)^2\right\rangle_{\rm FS}\left\langle\left(\mathbf{q}\cdot\mathbf{v}_{\mathbf{k}-}\right)^2\right\rangle_{\rm FS}}{\omega_{\rm G}^2(\mathbf{q})} \quad \text{(C.3)}$$

with the Fermi surface average $\langle \ldots \rangle$ defined as $\langle z(\mathbf{k}')\rangle_{\rm FS} = \int d\phi \int d\theta z(\mathbf{k}') \sin\theta$ for a given function $z(\mathbf{k}')$. Finally, assuming the same DoS on both bands, i.e. $N_\mu = N_0$, and almost similar Fermi velocities, i.e. $(v_{\rm F+} - v_{\rm F-}) \ll v_{\rm F}$ with $v_{\rm F} = \max_\mu v_{\rm F\mu}$, one obtains after performing the integration:

$$\frac{\omega_{\rm L}^2(\mathbf{q}) - \omega_{\rm L}'^2(\mathbf{q})}{v_{\rm F}^2|\mathbf{q}|^2} \approx \frac{1}{6}\frac{\left(v_{\rm F+}^2 - v_{\rm F-}^2\right)}{v_{\rm F+}^2 + v_{\rm F-}^2}\frac{\left(v_{\rm F+}^2 - v_{\rm F-}^2\right)}{v_{\rm F}^2} \ll 1 \tag{C.4}$$

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